Summary of PhD thesis

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The definition of dyadic derivative was introduced by Butzer and Wagner (1973). It has been generalised to Vilenkin groups by Onneweer. F. Schipp showed on the Walsh group that the dyadic derivate of the dyadic integral of an integrable function give back the function itself. This theorem was studied on the Vilenkin groups by P. Simon and J. Pál and on two-dimensional Walsh group by G. Gát. We proved on two-dimensional bounded Vilenkin groups the following:

Theorem *If $f \in L^1(G_m \times G_{\tilde{m}})$ and $\hat{f}(n_1, n_2) = 0$ whenever $n_1 n_2 = 0$, then $d_{\mathbf{n}}(\mathbf{I}f) \to f$ a.e., as $n_1, n_2 \to \infty$ in such a way that $|n_1 - n_2| \leq \beta$, where β is a fixed parameter.*

We define the operator T by

$$Tf := \sup_{n_1, n_2 \in \mathbf{N} \ |n_1 - n_2| \leq \beta} |d_{\mathbf{n}}(\mathbf{I}f)|.$$

During the proof of Theorem we used the following important lemma proved also.

Lemma *The operator T is of weak type $(1,1)$.*

We stated an application in the following Corollary.

Corollary *There exists a constant c for operator T such that*

$$\|Tf\|_1 \leq c\||f|\|_{H^\circ} \qquad (|f| \in H^\circ).$$

The Walsh system in the Kaczmarz enumeration was studied by a lot of authors. F. Schipp and Wo-Sang Young proved that the Walsh-Kaczmarz system is a convergence system. Skvorcov proved for continuous functions f, that Fejér means converges uniformly to the function f itself. G. Gát proved the a.e. convergence $\sigma_n f \to f$ $(n \to \infty)$ and he proved also that the maximal operator of Fejér means defined by $\sigma^* f := \sup_n |\sigma_n f|$ is of type (q,q) for all $1 < q \leq \infty$ and of weak type $(1,1)$. We generalized this results. We proved the following theorems on p-series field:

Theorem $\sigma_n^\chi f \to f$ $(n \to \infty)$ *almost everywhere for all $f \in L^1(G_p)$.*

Theorem *The operator $\sigma^{*\chi}$ is of type (q,q) for all $1<q\leq\infty$, weak type $(1,1)$ and of type (H^1,L^1).*

We remark the fact that the operator $\sigma^{*\chi}$ is of type (H^1,L^1) was not proved by G. Gát in the Walsh-Kaczmarz case.

G.I. Sunouchi introduced his operators T^ω and U^ω in 1951. He proved that T^ω and U^ω are of type (p,p) for all $1<p<\infty$. P. Simon proved that the operator T^ω is of type (H^1,L^1) and the operator U^ω is not of type (H^1,L^1). We proved the following properties of the Sunouchi operators with respect to Walsh-Kaczmarz system:

Theorem *The operator T^κ is of type (H^1,L^1).*

Theorem *The operator T^κ is of weak type $(1,1)$.*

Corollary *The operator T^κ is of type (p,p) $(1<p\leq 2)$.*

After this we showed that the Dirichlet kernels D_n^κ has of the following form:

Theorem *If $n\in\mathbf{P}$, then*

$$\begin{aligned} D_n^\kappa(x) &= n_{|n|}D^\omega_{2^{|n|}}(x)+r_{|n|}(x)\sum_{j=1}^{|n|}n_{|n|-j}\times \\ &\times\prod_{l=0}^{j-1}r_{|n|-l}^{n_{|n|-l}}(\tau_{|n|}(x))D^\omega_{2^{|n|-j}}(\tau_{|n|}(x)). \end{aligned}$$

Using this it is easy to see:

Corollary *The operator U^κ is not of type (H^1,L^1).*