The $n$-th Walsh-Paley function is $\omega_n(x) = (-1)^{\sum_{k=0}^{n-1} x_k}$, where $n = \sum_{k=0}^{\infty} x_k 2^k \in \mathbb{Z}$ and $x_k, n_k \in \{0, 1\}$, $n \geq 1$.

The Dirichlet and Fejér kernels are defined as

$$D_n(x) = \sum_{k=0}^{n-1} \sigma_k(\omega_{n-k}(x)), \quad K_n(x) = \frac{1}{n} \sum_{k=0}^{n-1} D_k(x).$$

Fourier coefficients, partial sums of Fourier series, Fejér means:

$$f(n) = \frac{1}{2} \int_{-1}^{1} f(x) \omega_n(x) dx, \quad S_m(f)(x) = \sum_{k=-m}^{m} f\left(\frac{k}{2m}\right),$$

$$S_m(f)(x) = \frac{1}{2} \sum_{k=-m}^{m} \omega_k \left(1 - \frac{k}{2m}\right) f\left(\frac{k}{2m}\right).$$

That is,

$$\left(\sigma_k \omega_n \right) = \left(\left(\sum_{k=0}^{n-1} \omega_k \omega_n \right) \frac{n}{2^{n+k+1}} \right) \quad \left(\left(\omega_k \omega_n \right) \frac{n}{2^{n+k+1}} \right) \quad \text{for all } n \in \mathbb{N}.$$

In other words, the Walsh-Paley and the Walsh-Kaczmarz systems are dyadic blockwise rearrangement of each other.

The $(C, \alpha)$ Cesàro means of the integral function $f$ are $\sigma^\alpha(f)(x) = \frac{1}{n} \sum_{k=0}^{n} S_k(f)(x)$, where $n \geq 1$.

Fourier coefficients and partial sums of Fourier series, Fejér means:

$$f(n) = \frac{1}{2} \int_{-1}^{1} f(x) \omega_n(x) dx, \quad S_m(f)(x) = \sum_{k=-m}^{m} f\left(\frac{k}{2m}\right).$$

The two-dimensional Fourier series is defined as

$$S_{m,n}(f)(x,y) = \sum_{k,l=-m}^{m} f(\frac{k}{2m},\frac{l}{2n}) \omega_k \omega_l(x,y).$$

Two-dimensional Fourier coefficients of two variable integrable functions $f : |x|, |y| \leq 1 \rightarrow C$ are

$$f(k,n) = \frac{1}{4mn} \int_{0}^{2\pi} \int_{0}^{2\pi} f(x,y) \omega_k(x) \omega_n(x) dx dy.$$