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dedicated to the
75th birthday of Professor Ferenc Schipp,
to the
70th birthday of Professor William Wade
and to the
65th birthday of Professor Péter Simon

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On absolute convergence of the series of Fourier coefficients with respect to Haar-like systems

Alexander Aplakov

Abstract

We study the absolute convergence of the series of Fourier coefficients with respect to Haar-like systems for functions of the class generalized bounded variation $BV(p (n) \uparrow p, \phi)$.

In 1999 T. Akhobadze has considered the $BV(p (n) \uparrow p, \phi)$ class of functions.

Let $\phi$ be an increasing function determined on the set of natural numbers. Moreover, $\phi(1) \geq 2$ and $
\lim_{n \to \infty} \phi(n) = +\infty$.

Let $f$ be a finite 1-periodic function defined on the interval $(-\infty, +\infty)$. $\Delta$ is said to be a partition with period 1 if there is a set of points $t_i$ for which

$\ldots t_{-1} < t_0 < t_1 < t_2 < \cdots < t_m < t_{m+1} < \ldots$,

and $t_{k+m} = t_k + 1$, when $k = 0, \pm 1, \pm 2, \ldots$, where $m$ is any natural number.

Definition 1  Let $p (n)$ be an increasing sequence such that $1 \leq p (n) \leq p, n \to \infty$, where $1 \leq p \leq \infty$. We say that a function $f$ belongs to class $BV(p (n) \uparrow p, \phi)$, if

$$V(f, p (n) \uparrow p, \phi) =$$

$$= \sup_{n \geq 1} \sup_{\Delta} \left\{ \left( \sum_{k=1}^{m} |f(t_k) - f(t_{k-1})|^p(n) \right)^{1/p(n)} : \rho(\Delta) \geq \frac{1}{\phi(n)} \right\} < \infty,$$

where $\rho(\Delta) = \min_k |t_k - t_{k-1}|$.

We note that if $p (n) = p$ for each natural number, where $1 \leq p (n) < \infty$, then the class $BV(p (n) \uparrow p, \phi)$ coincides with the Wiener class $V_p$. If $\phi(n) = 2^n, n = 1, 2, \ldots$, then the class $BV(p (n) \uparrow p, \phi)$ coincides with the class $BV(p (n) \uparrow p)$ introduced by H. Kita and K. Yoneda in 1990.

Denote by $a_m(f)$ the Fourier coefficients of the function $f \in L([0, 1])$ with respect to Haar-like systems, i.e.

$$a_m(f) = \int_0^1 f(t) \overline{\chi_m(t)} dt \quad (m = 1, 2, \ldots).$$

The present paper is devoted to the question of convergence of the series

$$\sum_{m=1}^{\infty} m^\alpha |a_m(f)|^\beta$$
in the class $BV(p(n) \uparrow p, \phi)$.

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Some properties of kernels with respect to Vilenkin-like systems

István Blahota

Abstract

The investigation of kernel functions is an important part of the Fourier analysis. $D_n$ and $K_n$ maximal values of the $n$-th Dirichlet and Fejér kernels for Walsh-Paley, Vilenkin and some other systems are $n$ and $\frac{n-1}{2}$, respectively. We deal with a more general system; in this case the situation is different. In our poster we show some results on this topic and a concrete example, when $D_n > n$ and $K_n > \frac{n-1}{2}$ for several $n \in \mathbb{P}$.

References


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Signals and systems theory and identification using rational orthogonal bases and special wavelet constructions

József Bokor, Alexandros Soumelidis and Zoltán Szabó

Abstract

Systems and Control Laboratory of the Institute for Computer Science and Control (Hungarian Academy of Sciences) since three decades now is recognized as one of the most significant bases of systems and control theory and related disciplines. Besides the theoretical research in several topics of this field, attention is paid on technical realizations and applications in various fields in industry, energy production, and vehicles technology too. An undoubtedly fruitful cooperation has been formed by the Lab members (including the authors of this lecture) and Prof. Ferenc Schipp over the years; this lecture offers an excellent opportunity to review and give a short summary of some specialities of this activity, and the result obtained.

Applying non-standard bases in system identification received specific attention in the 90's since as a requirement of applying robust control methods related to signal and systems representations in the Hardy space \( H^\infty \). Rational orthogonal bases have been obtained by using the concept of the Malmquist-Takenaka system, and proved to be useful tool of representing signals and systems in the cases when the poles associated with them are — at least approximately — known. Efficient numeric algorithms have also been worked out that facilitate the use of these bases in practical problems.

In cases when no \textit{a priori} information is available rational orthogonal bases do not solve the general problem of system identification. Applying the concept of the voice-transform on group representations in the space \( H^2 \) a wavelet-like construction — that is related to the hyperbolic geometry of the unit disc in the complex plane — has been worked out that seemed to be a promising tool of finding system poles. A special — and computable — form of the hyperbolic wavelet can be obtained starting from the Laguerre-system.

On the basis of the Laguerre system, using the hyperbolic nature of the geometry in the unit disc, a method of finding the poles of a system has been elaborated. This method that does not require any \textit{a priori} assumption of either the pole locations or the number and multiplicity of them, hence it can be considered as a non-parametric solution of the system identification problem. Finding the ways of practical applications is the subject of current research.

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Discrete orthogonality of the analytic wavelets in the Hardy space of the upper half plane

Tímea Eisner and Margit Pap

Abstract
We will prove that the analytic orthogonal wavelet-system, which was introduced by H. G. Feichtinger and M. Pap in [2] is discrete orthogonal too.

We will discuss the discrete orthogonality and the properties of the reproducing kernel functions of the introduced wavelet-spaces.

References


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Discrete wavelets in Walsh analysis

Yuri Farkov

Abstract

We give a review of recent results on discrete wavelets in Walsh analysis. In [1], the Vilenkin-Chrestenson transform is employed in spaces of complex periodic sequences to define analogues of orthogonal wavelets, which have been previously studied for the case of Cantor and Vilenkin groups. The papers [2] and [3] contain examples of the use of these wavelets to signal and image processing. Among the main subjects to be discussed are algorithms to construct biorthogonal wavelets associated with the generalized Walsh functions. Some new examples of wavelets in the spaces of periodic complex sequences will be presented (cf. [4], [5] and references therein). In constructing discrete periodic wavelets the specific features of the finite dimensional case are revealed not only in the simplification of proofs, but also in having more freedom in choosing values of parameters. Indeed, in the constructions of wavelets proposed in [1] and [5], the condition of absence of blocking sets is insignificant: ”degenerate” families of parameters for wavelets on the Cantor and Vilenkin groups give wavelet bases in sequence spaces (see [6, Theorem 3.4] for a similar phenomenon in frame constructions).

References


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Hörmander-Mihlin multipliers in Hardy spaces

Sándor Fridli

ABSTRACT

In this talk we are concerned with Hörmander-Mihlin multipliers [1-4]. They can be viewed as natural generalizations of the classical Marcinkiewicz [6] multiplier conditions which are known to be sufficient for the corresponding multiplier operator be bounded on $L^p_{2\pi}$ provided $p > 1$. We show that for Hörmander-Mihlin [5], [7] multipliers the scale of Hardy spaces is a more proper choice than that of the Lebesgue spaces. Both the trigonometric and the dyadic versions of the problem will be addressed. We note that these conditions apply directly to the sequence generating the multiplier operator rather than to the corresponding kernel function.

REFERENCES


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Summation methods of Walsh-Fourier series, almost everywhere convergence and divergence

György Gát

Abstract

Let $x$ be an element of the unit interval $I := [0, 1)$. The $n$-th Walsh function is

$$\omega_n(x) := (-1)^{\sum_{k=0}^{\infty} n_k x_k} \quad (n = \sum_{k=0}^{\infty} n_k 2^k, \ x = \sum_{k=0}^{\infty} \frac{x_k}{2^k+t} \in I).$$

The $n$-th Walsh-Fourier coefficient, the $n$-th partial sum of the Fourier series, the $n$-th $(C,1)$ mean of $f \in L^1(I)$:

$$\hat{f}(n) := \int_I f(x) \omega_n(x) dx, \quad S_nf := \sum_{k=0}^{n-1} \hat{f}(k) \omega_k, \quad \sigma_nf := \frac{1}{n} \sum_{k=0}^{n-1} S_kf.$$

Fine proved [1] that for each integrable function we have the almost everywhere convergence of Fejér means $\sigma_nf \to f$. In the talk we give a brief résumé of the recent results with respect to almost everywhere summability of Walsh-Fourier series of one and two dimensional functions. With respect to the two-dimensional issue some convergence properties of the Marczinkiewicz means and its generalizations are also discussed. The Marcinkiewicz means [2] are defined as

$$t_nf(x) := \frac{1}{n} \sum_{k=0}^{n-1} S_{k,k}f(x).$$

References


ECG-based heart beat detection using rational functions

Zoltán Gilián

ABSTRACT

The electrocardiogram (ECG) is the most common diagnostic tool in cardiology. Since heart beat locations carry important diagnostic information, heart beat detection is an essential task in ECG signal processing.

The aim of this talk is to present a novel heart beat detection algorithm using rational modelling of ECG signals. The algorithm considers several candidate beat locations. For a given candidate a rational model is fitted to the ECG signal by means of numerical optimization and Fourier partial sums with respect to the Malmquist-Takenaka system. The resultant model parameters are used as a basis of classification. The classification is performed by an SVM classifier, which is trained on annotated ECG records of the PhysioNet database.

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On the strong summability of Walsh-Fourier series

Ushangi Goginava

Abstract

Almost everywhere strong \( \Phi \)-summability of one-dimensional Fourier series in Walsh and trigonometric systems established by Rodin and Schipp. We proved that if the growth of a function \( \Phi \) is bigger than the exponential, then the strong \( \Phi \)-summability of a Walsh-Fourier series can fail everywhere.

It is proved that of the Marcinkiewicz type of two-dimensional Walsh-Fourier series of the continuous function \( f \) is uniformly strong summable to the function \( f \) exponentially in the power \( 1/2 \). Moreover, it is proved that this result is best possible.

We proved the a. e. relation

\[
\frac{1}{n} \sum_{m=0}^{n-1} |S_{m,m}f - f|^q \to 0
\]

for every two-dimensional function belonging to \( L \log L \) and \( q > 0 \).

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Construction of positively invariant sets for initial value problems using Fourier series

Zoltán Horváth

Abstract

For physically reliable numerical computations for time dependent differential equations modelling, e.g. heat and material transport it is very important to find discrete positively invariant subsets of the state space and determine time step sizes of the numerical methods that guarantee this.

In this talk first we present the results of our general theorem which provides a computable formula for the largest step size guaranteeing discrete positive invariance. However, in certain "smooth" situations this threshold is not sharp. To capture sharp results in smooth situations we construct positively invariant and attractive sets with the help of Fourier series. We remark that this topic is closely related to the analysis of positivity of trigonometric series. Finally, we show numerical experiments that demonstrate our findings.

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Quantum computers and Fourier transform

Antal Járai

ABSTRACT

In this short overview we consider connection of the (planned) quantum computers and Fourier transform and a basic application possibility for factorization of large numbers.

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On the divergence sets of some sequences of operators

Grigori Karagulyan

Abstract

The problem of characterization of divergence sets of power series, series in different classical orthonormal systems, as well as extremal sets of analytic and harmonic functions is considered in variety of papers. For the power and trigonometric series this problem was considered by Lusin, Hardy, Littlewood, Steinhaus, Erdős, Piranian, Herzog, Dvoretzky, Neder, Stechkin, Zeller, Taykov, Kahane, Katznelson, Buzdalin. For the series in other classical orthogonal systems it is considered by Wade, Simon, Harris, Kheladze, Lukashenko, Lunina, Prokhorenko, Bugadze, Goginava. The problem of characterization of boundary extremal sets of analytic and harmonic functions are considered by Lohwater, Piranian, Dolzhenko, Herzog. Reviews of some of these problems may be found in survey articles by Ul’yanov, Wade and in the monograph of Collingwood and Lohwater.

We consider this problem in general settings. Let $M[0,1]$ be the Banach space of bounded functions and $U_n(x,f) : L^1[0,1] \rightarrow M[0,1]$ be a sequence of bounded linear operators possessing the properties: 1) $\varrho = \sup_n \|U_n\|_{L^\infty\rightarrow M} < \infty$, 2) if $f \in M[0,1]$ and $f(x) = c$, $x \in (\alpha,\beta)$, then $U_n(x,f)$ converges uniformly in each closed subsets of the interval $(\alpha,\beta)$, 3) for any function $f \in L^\infty[0,1]$ the sequence $U_n(x,f)$ converges almost everywhere.

Theorem 1 ([1]) If a sequence of operators satisfies the conditions 1)-3), then for the set $E \subset [0,1]$ to be the divergence set of a sequence $U_n(x,f)$ for some function $f \in L^\infty[0,1]$, it is necessary and sufficient to be $G_{\delta\sigma}$-null set.

Typical examples of such operator sequences are: a) partial sums operators of Fourier series in Haar and Franklin systems, b) $(C,\alpha)$-means of Fourier series in trigonometric and Walsh systems with $\alpha > 0$ and c) Poisson integrals in unit ball. So this theorem gives a complete characterization of divergence sets of these operators. Some other related problems will be discussed too ([2]).

References


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Discrete orthogonal and biorthogonal product systems

Balázs Király

Abstract

By the generalization of the definition of the product system we give a method to construct orthogonal systems or biorthogonal system-pairs. These systems have several useful properties. With these systems we can define efficient interpolation algorithms and the Fourier coefficients with respect to these systems can be computed with an FFT-like algorithm.

References


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Compressing biomedical signals by using rational functions

Péter Kovács

Abstract

The analysis of physiological signals by means of mathematical transforms proved to be an effective method in various aspects. In this talk we consider the compression of these signals by using different types of rational systems [1, 2]. There is a wide range of applications which assess the same problem by employing B-splines [3], orthogonal polynomials [4], principal component analysis, wavelet transforms [5], etc. In our talk we give a brief introduction to these algorithms along with the recent methods based on rational function systems. The main advantage of these systems over the previous transformations is the adaptive behaviour due to the free parameters, i.e., the poles. We note that the optimization of these free parameters of the rational functions results in a compact representation of the signal. In previous works the number of the poles were fixed, and optimization algorithms were applied to the positions of the poles only [6, 7]. In this talk we present a new optimization method that applies for both the optimal positions and the number of the poles. Examples for practical applications including ECG and EEG signal processing [8, 9], will be provided.

References


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Uncertainty principle for the dyadic group

Aleksander V. Krivoshein* and Elena A. Lebedeva†

ABSTRACT

We introduce a notion of localization for dyadic functions, i.e. functions defined on the Cantor group. Localization is characterized by a functional $UC_d$ that is similar to the Heisenberg uncertainty constant for real-line functions.

**Definition 1** Suppose $(\mathbb{R}_+, \oplus)$ is a representation of the Cantor group, $f \in L^2(\mathbb{R}_+)$ is a complex valued dyadic function, and $\hat{f}$ is its Walsh-Fourier transform, then the functional

$$UC_d(f) := V(f)V(\hat{f}),$$

where

$$V(f) := \frac{1}{\|f\|_{L^2(\mathbb{R}_+)}^2} \min_x \int_{\mathbb{R}_+} (x \oplus \tilde{x})^2 |f(x)|^2 dx,$$

$$V(\hat{f}) := \frac{1}{\|\hat{f}\|_{L^2(\mathbb{R}_+)}^2} \min_t \int_{\mathbb{R}_+} (t \oplus \tilde{t})^2 |\hat{f}(t)|^2 dt$$

is called the dyadic uncertainty constant (the dyadic UC) of the function $f$.

We suggest a dyadic analog of quantitative uncertainty principle.

**Theorem 1** For any function $f \in L^2(\mathbb{R}_+)$, the following inequality holds

$$UC_d(f) \geq C,$$

where $C \simeq 8.5 \times 10^{-5}$.

To justify our definition we use some test functions including dyadic scaling and wavelet functions.

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Walsh and wavelet methods for differential equations on the Cantor group

Elena A. Lebedeva* and Maria Skopina

Abstract

Ordinary and partial differential equation for unknown functions defined on the Cantor dyadic group are studied. We consider two types of equations: related to the Gibbs derivatives and to the fractional modified Gibbs derivatives (or pseudo differential-operators). We find solutions in classes of distributions and study under what assumptions these solutions are regular functions with some "good" properties.

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Dyadic Blaschke products and reciprocal Blaschke functions

Levente Lócsi

Abstract

When we form compositions of dyadic Blaschke products, i.e. two-factor products of Blaschke functions of the form

\[ B_a: \bar{\mathbb{D}} \rightarrow \mathbb{D}, \quad B_a(z) = \frac{z - a}{1 - \bar{a}z}, \]

where \( \bar{\mathbb{D}} \) denotes the closed complex unit disk and \( a \in \mathbb{D} \), then the zeros of the composition can be calculated by solving polynomial equations. (The composition is also a Blaschke product with an additional unitary factor.) Such constructions can be used to define FFT algorithms for rational function systems [3].

Now we shall investigate the inverse problem. Can we find the parameters of for the Blaschke products to compose, when we are given the zeros of a composition? The most simple case with forming the composition of two two-factor Blaschke products is considered. Interestingly many answers come in the form of Blaschke functions, and furthermore reciprocal Blaschke functions can be introduced [1].

Similar calculations may lead to further interesting novelties related to the intersection properties of argument functions, constructions in the Poincaré disk model of hyperbolic geometry, and the hyperbolic analogue of the Apollonian circles.

Examples in MATLAB will be also presented [2].

References


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Integrability of the maximal function of Fejér kernel

Nacima Memić

Abstract
We study local integrability of partial maximal functions obtained from Walsh-Kaczmarz system. An estimate for the integral of the maximal function of Fejér kernel related to Walsh-Paley system is also stated.

References

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Statistical limit of Lebesgue measurable functions at $\infty$ with applications in Fourier analysis and summability

Ferenc Móricz

Abstract

This is basically a survey paper on recent results indicated in the title. A function $s : [a, \infty) \to \mathbb{C}$, measurable in Lebesgue’s sense, where $a \geq 0$, is said to have statistical limit $\ell$ at $\infty$ if for every $\varepsilon > 0$,

$$\lim_{b \to \infty} (b - a)^{-1}|\{\nu \in (a, b) : |s(\nu) - \ell| > \varepsilon\}| = 0.$$ 

We briefly summarize the main properties of this new concept of statistical limit at $\infty$. Then we demonstrate its applicability in Fourier Analysis. For example, the classical inversion formula involving the Fourier transform $\hat{s}$ of a function $s \in L^1(\mathbb{R})$ remains valid even in the general case when $\hat{s} \notin L^1(\mathbb{R})$. We also present Tauberian conditions, under which the ordinary limit of a function $s \in L^1_{\text{loc}}[1, \infty)$ follows from the existence of the statistical limit of its logarithmic mean at $\infty$.

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On the summability by Riesz method of series with respect to block-orthonormal systems

Givi Nadibaidze

Abstract

Below a question connected with the problems of almost everywhere summability by Riesz methods of series with respect to block-orthonormal systems are considered.

Let \( \{N_k\} \) be increasing sequences of natural numbers and \( \Delta_k = (N_k, N_{k+1}] \), \( k \geq 1 \). Let \( \{\varphi_n\} \) be a system of functions from \( L^2(0,1) \). The system \( \{\varphi_n\} \) will be called a \( \Delta_k \)-orthonormal system if \( ||\varphi_n||_2 = 1, n = 1, 2, ... \) and \( (\varphi_i, \varphi_j) = 0 \), for \( (i,j) \in \Delta_k, i \neq j, (k \geq 1) \).

Some results connected with the a. e. convergence and summability of series with respect to block-orthonormal systems are presented in [2], [3], [4].

Let \( \{\lambda_n\} \) be increasing sequences of positive numbers, \( \lambda_0 = 0 \) and \( \lambda_n \to +\infty \). The series \( \sum_{n=1}^{\infty} u_n \) is called summable by Riesz \( (R, \lambda_n, 1) \) method to the number \( s \) if

\[
\lim_{n \to \infty} \sum_{k=0}^{n} \left( 1 - \frac{\lambda_k}{\lambda_{n+1}} \right) u_k = s.
\]

It is established the conditions on \( \{N_k\} \) when the sequences \( \{ (\log_2 \log_2 \lambda_n)^2 \} \) guarantees the \( (R, \lambda_n, 1) \) summability a.e. of series \( \sum_{n=1}^{\infty} a_n \varphi_n(x) \) with respect to any \( \Delta_k \)-orthonormal system \( \{\varphi_n\} \). Note, that the sequences \( \{ (\log_2 \log_2 \lambda_n)^2 \} \) is the exact (in certain sense) Weyl multiplier for the \( (R, \lambda_n, 1) \) summability a.e. of series with respect to any orthonormal system (see [1]).

References


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Restricted two-dimensional Walsh-Fejér means and generalizations

Károly Nagy*

Abstract

Gát gave a common generalization of the result of Marcinkiewicz and Zygmund [4] and the result of Jessen, Marcinkiewicz and Zygmund [3]. He gave a necessary and sufficient condition for cone-like sets in order to preserve this convergence property [1] for two-dimensional Fejér means of trigonometric Fourier series. In 2010, the analogue of this result for Walsh system was discussed by Gát and Nagy [2].

In our poster we show some new results written in the last years by the author and others with respect to general orthogonal systems, Vilenkin systems [5, 7, 8], representative product systems [5] and Vilenkin-like systems [6].

References


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27
Discretization of the voice transforms of the Blaschke group

Margit Pap

Abstract

In papers [4, 7, 8, 9] joint with F. Schipp we have started to study the properties of the continuous voice transforms generated by representations of the Blaschke group on the Hardy space of the unit disc and the weighted Bergman spaces respectively. Analyzing the question of discretization of these voice transforms it turned out that different techniques are required. In the case of some weighted Bergman spaces the unified approach of atomic decomposition, developed by Feichtinger and Gröchenig, can be applied because the square integrability and the integrability condition are satisfied [5]. In this way we proved that not only the characteristic function of the unit disc will generate atomic decomposition, but every function from the minimal Möbius invariant space will generate an atomic decomposition in those weighted Bergman spaces. In the case of the voice transforms generated by the representations of the Blaschke group on Bergman spaces and the Hardy space of the unit disc respectively, the unified approach of atomic decomposition can not be applied. In the case of the Bergman space the voice transform is square integrable, but not integrable. In the case of the Hardy space even the square integrability is not satisfied. In both cases in order to discretize the voice transforms we introduced multiresolution analysis in these spaces. This constructions lead us to the construction of analytic wavelets [1, 3, 6].

References


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(C, 1)-summability and divergence of rearranged Walsh-Fourier series

Igor Polyakov*

Abstract

This lecture is devoted to review some recent results on constructing diverging rearranged Walsh-Fourier series. For example, there exists \( f \in L_0(\ln^+)L \) for which Walsh-Kaczmarz-Fourier series diverge almost everywhere. The proof of this theorem is based on

Lemma 1 For each positive and increasing sequence \( \{\lambda_n\} \), such that

\[
\sum_{n=1}^{\infty} \frac{1}{n \lambda_n} = \infty,
\]

it is true that almost everywhere in \([0, 1]\)

\[
\limsup_{n \to \infty} \frac{D_n^\lambda(x)}{\lambda_n} = \infty.
\]

We also have an example of piece-wise linear rearrangement of Walsh-Paley system and \( f \in L \) for which \((C, 1)\)-means of rearranged Walsh-Fourier series diverge almost everywhere.

References


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Extremal problems for positive definite functions on groups

Szilárd Révész

Abstract

This paper is a survey of recent results about various Fourier analysis extremal problems, which were extended to locally compact Abelian groups (LCA groups) and even to some locally compact groups (lc groups) lacking commutativity.

To formulate the general framework, the starting point is the notion of positive definite functions, which can be considered on any groups (even if not topological). We consider a subset $\Omega$ of the group $G$, and the class of positive definite functions, which vanish outside $\Omega$ and fall into a reasonable class of "nice" functions, like e.g. continuous, bounded measurable, or integrable functions. With respect to a function $f$ vanishing outside $\Omega$, also a number of different variations can be considered, from just $f(x) = 0 (\forall x \notin \Omega)$ to $\text{supp } f \subseteq \Omega$. The classes of real-valued and complex-valued positive definite functions can both be considered. Since in general $\sup |f| = f(0)$ for any positive definite function $f$, it is natural to normalize by assuming $f(0) = 1$.

Let $\mathcal{F}_G(\Omega)$ denote one of these possible classes of (normalized) positive definite functions. A number of classically investigated extremal problems can be formulated or reformulated in quite general terms say on lc groups by posing the maximization problem of some functional on (some of the possible) classes $\mathcal{F}_G(\Omega)$. In particular, we will discuss the problem of maximizing the integral $\int_G f d\mu_G$ (with $\mu_G$ the Haar measure of $G$) – often called "Turán type extremal problem", but in fact going back to at least the thirties and to Siegel – and the problem of maximizing the point function value $|f(z)|$ (with of course $z \in \Omega$, if we want a nontrivial answer) – sometimes called "pointwise Turán problem" but in fact traced back to a century and better named "Carathéodory-Fejér type extremal problem".

We would also like to call attention to some rather general new notions, like asymptotic uniform upper density on LCA groups and round elements of lc groups. These occurred in our investigations naturally, but their context may be useful for dealing with other analysis problems on groups.

We will mention already published and (yet) unpublished results, part of which are joint work with S. Krenedits.

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Maximal convergence space of the \((C, \alpha)\) means of two-dimensional integrable functions on the 2-adic additive group

György Gát* and Ilona Simon†

Abstract

In this lecture In this paper we investigate Cesàro summability of two-dimensional integrable functions on the 2-adic group. We prove the a.e. convergence of 2-adic Cesàro means \(\sigma_{n,m}^{\alpha,\beta} f \to f\) as \(n,m \to \infty\) for functions \(f \in L^{\log^+}L(I_2)\) and \(\alpha, \beta > 0\).

Then we show, that this convergence result can not be improved in the Pringsheim sense, that is, \(L^{\log^+}L\) is the maximal convergence space for \(\sigma_{n,m}^{1,1}\) when there aren’t conditions for the indices except that they tend to infinity.

We prove, that for all measurable functions \(\delta : [0, \infty) \to [0, \infty)\) for which \(\lim_{t \to \infty} \delta(t) = 0\) there is a function \(f \in L^{\log^+}L\delta(L)\) with \(\lim \sup |\sigma_{n_1, n_2} f(x)| = +\infty\) a.e. as \(\min\{n_1, n_2\} \to \infty\).

References


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Remarks on methods to compute dyadic derivatives on finite groups

Radomir S. Stanković

Abstract

In this paper, we discuss few methods for computing dyadic derivatives on finite Abelian groups. We present methods based on vectors and decision diagrams as underlying data structures to represent functions to be processed. The considered methods use different properties of dyadic derivatives to improve the efficiency of computation on Central Processing Units (CPUs) and Graphics Processing Units (GPUs). Experimental results comparing the implementation of these methods and related algorithms are presented.

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On various types of continuity of multiple dyadic integrals

Valentin Skvortsov*

Abstract

It is known that the problem of recovering the coefficients of multidimensional Haar and Walsh series from their sums by generalized Fourier formulas can be reduced to the one of recovering a function (the so-called quasi-measure) from its derivative with respect to the appropriate dyadic derivation basis, which in turn can be solved by the choice of a suitable integration process. The choice of a basis depends on the type of convergence. The difficulties which should be overcome in applying this method are related to the fact that the primitive we want to recover is differentiable not everywhere but outside an exceptional set which is not countable in a dimension greater than one.

We investigate continuity assumptions which should be imposed on the primitive at the points of exceptional sets to guarantee its uniqueness. The most natural integration process to recover the primitive is Kurzweil-Henstock integral. We consider continuity properties of the dyadic Kurzweil-Henstock integral. It turns out that this integral which solves the problem in the one-dimensional case, is not strong enough to recover the primitive in multidimensional case under our assumptions on the exceptional sets. Because of this we have to introduce suitable Perron-type integrals defined by major and minor functions having special continuity properties. The type of continuity is implied by the type of convergence of multiple Haar or Walsh series, for which we want to solve the coefficients problem.

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Variable Lebesgue spaces and continuous wavelet transforms

Kristóf Szarvas

Abstract

The so-called variable Lebesgue spaces are studied intensively in the last few years. Instead of the classical $L^p$-norm, the variable $L_{p(\cdot)}$-norm is defined by the Luxemburg-norm. The variable Lebesgue spaces have a lot of common property with the classical Lebesgue spaces, for example completeness, duality, embedding, boundedness of the classical Hardy-Littlewood maximal operator (see Cruz-Uribe and Fiorenza [1], Diening, Hästö and Růžička [2]). But there are some properties, which hold on the classical Lebesgue spaces, but do not hold on the variable Lebesgue spaces (for example translation invariance, boundedness of the strong maximal operator, Young’s inequality and rearrangement invariance).

The variable Lebesgue spaces are special cases of more general function spaces, the so-called Musielak-Orlicz spaces. The classical Lebesgue spaces, the weighted Lebesgue spaces and the weighted variable Lebesgue spaces can also be viewed as Musielak-Orlicz spaces.

The inversion formula for the continuous wavelet transform holds for all $f \in L_2(\mathbb{R}^d)$. Under some conditions

$$\lim_{S \to 0} \int_{S}^{\infty} \int_{\mathbb{R}^d} W_g f(x,s) T_x D_s \gamma \frac{dx ds}{s^{d+1}} = C_{g,\gamma} \cdot f,$$

where $W_g f$ is the continuous wavelet transform of $f$ with respect to a wavelet $g$ and $C_{g,\gamma}$ is a constant depending on $g$ and $\gamma$, but independent of $f$. The convergence holds in $L_p$-norm, almost everywhere and at each Lebesgue points for all $f \in L_p(\mathbb{R}^d)$ ($1 < p < \infty$) (see Weisz [3]).

In this paper we will investigate the norm- and almost everywhere convergence of the inversion formula in the $L_{p(\cdot)}$ spaces.

References


On the Fejér means of Walsh-Fourier series

Georgi Tephnadze

Abstract
Weisz [5] proved that maximal operator $\sigma^* f := \sup_{n \in \mathbb{N}} |\sigma_n f|$ with respect to Walsh system is bounded from the Hardy space $H_{1/2}$ to the space weak-$L_{1/2}$. The counterexample, which shows that boundedness does not hold when $p = 1/2$ due to Goginava [2] (see also [1] and [3]). Moreover, in [4] it was proved that there exist a martingale $f \in H_p$ ($0 < p \leq 1/2$), such that

$$\sup_n \|\sigma_n f\|_p = +\infty.$$ 

On the other hand, Weisz [6] proved that the maximal operator

$$\sigma^# f := \sup_{n \in \mathbb{N}} |\sigma_{2n} f|$$ 

is bounded from the martingale Hardy space $H_p$ to the space $L_p$ for $p > 0$.

This lecture is devoted to review the boundedness of subsequences of Fejér means of Walsh-Fourier series on the Hardy spaces, when $0 < p \leq 1/2$. In the talk will be presented necessary and sufficient conditions for the convergence of Fejér means in the terms of modulus of continuity of the Hardy spaces and convergence theorems of Walsh-Fejér means.

References

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Convergence of trigonometric and Walsh-Fourier series

Ferenc Weisz

Abstract

In this paper we present some results on convergence and summability of one- and multi-dimensional trigonometric and Walsh-Fourier series. The Fejér and Cesàro summability methods are investigated. We will prove that the maximal operator of the summability means is bounded from the corresponding classical or martingale Hardy space $H_p$ to $L_p$ ($p > p_0$). For $p = 1$ we obtain a weak type inequality by interpolation, which ensures the almost everywhere convergence of the summability means.

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Characterization of fractal functions in Hölder-type spaces on local fields

Su Weiyi

ABSTRACT

The characterization of fractal functions defined on local fields in Hölder-type space $C^\sigma(K_p)(\sigma > 0)$ are studied, and the equivalent theorem (Jackson theorem and Bernstein theorem) is shown: for $s \geq 0$, we have

$$f^{(s)} \in \text{Lip} (C^\sigma(K_p), \alpha), \alpha > 0 \iff E_p^n (C^\sigma(K_p), f) = O \left( p^{-n(\alpha+s)} \right), \ n \to +\infty.$$ 

Then the necessary preparation for the study of fractal PDE are given.

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