Matematika II. 3. feladatsor

Határozza meg az alábbi függvények deriváltjait

1) $f(x)=6x^5+4x^4-3x^2+2x+1$ $(x\in\mathbb{R})$; $f(x)=\frac{1}{x}+\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}$ $(x>0)$;
$f(x)=\sqrt[5]{x^2}-\frac{3}{x^2}+\sqrt{x}+\sqrt[3]{x}$ $(x>0)$; $f(x)=\sqrt{x^3\sqrt{x}}+\frac{2}{x\sqrt{x}}$ $(x>0)$;
$f(x)=x^2(\sqrt{x}-3x^2)$ $(x>0)$; $f(x)=5^x+3\lg x-2\ln x$ $(x>0)$;
$f(x)=2\cos x-3e^x-\pi^x$ $(x\in\mathbb{R})$; $f(x)=3\,\mathrm{sh}\,x+2\sin x+5\,\mathrm{ch}\,x$ $(x\in\mathbb{R})$;
$f(x)=2\,\mathrm{tg}\,x-3\,\mathrm{ctg}\,x$ $(x\in]0,\frac{\pi}{2}[)$; $f(x)=2\arcsin x-3\arccos x$;

2) $f(x)=(2x^2+3x+2)(5x^4+3x^2-1)$ $(x\in\mathbb{R})$; $f(x)=(3-5x)(2+x^2)$ $(x\in\mathbb{R})$;
$f(x)=2x(3x+2)(4x-3)$ $(x\in\mathbb{R})$; $f(x)=(1-x)(1-x^2)(1-x^3)$ $(x\in\mathbb{R})$;
$f(x)=x\,e^x$ $(x\in\mathbb{R})$; $f(x)=(x^2+2x+1)e^x$ $(x\in\mathbb{R})$; $y=x\ln x$ $(x>0)$;
$f(x)=x^5\cdot 5^x$ $(x\in\mathbb{R})$; $f(x)=\sqrt{x}\sin x$ $(x\geq 0)$;
$f(x)=(3x^2+2)\,\mathrm{arctg}\,x$ $(x\in\mathbb{R})$; $f(x)=\frac{x^2}{2}\left(\ln x-\frac{1}{3}\right)$ $(x>0)$;

3) $f(x)=\frac{6x+3}{4x-2}$ $(x\neq\frac{1}{2})$; $f(x)=\frac{x^2+3x-5}{x^2+x+1}$ $(x\in\mathbb{R})$; $f(x)=\frac{\ln x}{x}$ $(x>0)$;
$f(x)=\frac{1+e^x}{\sqrt[3]{x}+2^x}$ $(x>0)$; $f(x)=\frac{(3x^2-5)\cos x}{\sin x}$ $(x\neq k\pi)$; $f(x)=\frac{x\ln x}{e^x}$ $(x>0)$;
$f(x)=\frac{\mathrm{tg}\,x}{x^2+1}$ $(x\neq k\frac{\pi}{2})$; $f(x)=\frac{e^x+\sin x}{x\,e^x}$ $(x\neq 0)$;

4) $f(x)=(2x+5)^{20}$ $(x\in\mathbb{R})$; $f(x)=(x^2+x+1)^6$ $(x\in\mathbb{R})$; $f(x)=e^{e^x}$ $(x\in\mathbb{R})$;
$f(x)=e^{x^2+5x+1}$ $(x\in\mathbb{R})$; $f(x)=e^{x\ln x}$ $(x>0)$; $f(x)=\ln(\ln x)$ $(x>1)$;
$f(x)=\sin^2 x$ $(x\in\mathbb{R})$; $f(x)=\sin 3x\cos 5x$ $(x\in\mathbb{R})$;
$f(x)=\log_3(x^2+1)$ $(x\in\mathbb{R})$; $f(x)=\mathrm{arctg}\frac{x}{x^2+1}$ $(x\in\mathbb{R})$;
$f(x)=\sin^3(5x+4)$ $(x\in\mathbb{R})$; $f(x)=2^{\cos(x^2)}$ $(x\in\mathbb{R})$;

5) $f(x)=(\sin x)^{\cos x}$; $f(x)=x^x$ $(x>0)$; $f(x)=x^{\sqrt{x}}$ $(x>0)$;
$f(x)=x^{\sin x}$ $(x>0)$; $f(x)=(\ln x)^{x^2}$ $(x>1)$;

6) $f(x)=x^k$ $(x\in\mathbb{R})$, $f^{(n)}(x)=?$; $f(x)=\frac{1}{x}$ $(x\neq 0)$ $f^{(n)}(x)=?$;
$f(x)=\ln x$ $(x>0)$, $f^{(n)}(x)=?$; $f(x)=(x^2+3x+1)\sin x$ $(x\in\mathbb{R})$, $f^{(10)}(x)=?$

Matematika II.

Elemi függvények differenciálhányados függvényei

$f(x) = c \quad (x \in \mathbb{R}) \Rightarrow \exists\ f'(x) = 0 \quad (x \in \mathbb{R}) \qquad (c' = 0)$

$f(x) = x \quad (x \in \mathbb{R}) \Rightarrow \exists\ f'(x) = 1 \quad (x \in \mathbb{R}) \qquad ((x)' = 1)$

$f(x) = x^n \quad (x \in \mathbb{R}) \Rightarrow \exists\ f'(x) = n x^{n-1} \quad (x \in \mathbb{R}) \qquad ((x^n)' = n x^{n-1})$

$f(x) = \sum_{n=0}^{\infty} a_n x^n \quad (x \in]-s, s[) \Rightarrow \exists\ f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1}$

$f(x) = \exp(x) = e^x \quad (x \in \mathbb{R}) \Rightarrow \exists\ f'(x) = \exp(x) = e^x \qquad (\exp'(x) = \exp(x), (e^x)' = e^x)$

$f(x) = \sin(x) \quad (x \in \mathbb{R}) \Rightarrow \exists\ f'(x) = \cos(x) \quad (x \in \mathbb{R}) \qquad (\sin'(x) = (\sin(x))' = \cos x)$

$f(x) = \cos(x) \quad (x \in \mathbb{R}) \Rightarrow \exists\ f'(x) = -\sin(x) \quad (x \in \mathbb{R}) \qquad (\cos'(x) = (\cos(x))' = -\sin(x))$

$f(x) = \operatorname{sh}(x) \quad (x \in \mathbb{R}) \Rightarrow \exists\ f'(x) = \operatorname{ch}(x) \quad (x \in \mathbb{R}) \qquad (\operatorname{sh}'(x) = (\operatorname{sh}(x))' = \operatorname{ch}(x))$

$f(x) = \operatorname{ch}(x) \quad (x \in \mathbb{R}) \Rightarrow \exists\ f'(x) = \operatorname{sh}(x) \quad (x \in \mathbb{R}) \qquad (\operatorname{ch}'(x) = (\operatorname{ch}(x))' = \operatorname{sh}(x))$

$f(x) = \exp_a(x) = a^x \quad (x \in \mathbb{R}) \Rightarrow \exists\ f'(x) = \exp_a(x) \ln a = a^x \ln a \quad (x \in \mathbb{R}) \qquad ((\exp_a'(x) = (a^x)' = a^x \ln a)$

$f(x) = \log_a(x) \quad (x \in \mathbb{R}_+) \Rightarrow \exists\ f'(x) = \frac{1}{x \ln a} \quad (x \in \mathbb{R}_+) \qquad (\log_a'(x) = (\log_a(x))' = \frac{1}{x \ln a})$

$f(x) = \ln(x) \quad (x \in \mathbb{R}_+) \Rightarrow \exists\ f'(x) = \frac{1}{x} \quad (x \in \mathbb{R}_+) \qquad (\ln'(x) = (\ln(x))' = \frac{1}{x})$

$f(x) = x^{\mu} \quad (x \in \mathbb{R}_+) \Rightarrow \exists\ f'(x) = \mu x^{\mu-1} \quad (x \in \mathbb{R}_+) \qquad ((x^{\mu})' = \mu x^{\mu-1})$

$f(x) = \sqrt{x} \quad (x \in \mathbb{R}_+) \Rightarrow \exists\ f'(x) = \frac{1}{2\sqrt{x}} \quad (x \in \mathbb{R}_+) \qquad ((\sqrt{x})' = \frac{1}{2\sqrt{x}})$

$f(x) = \operatorname{tg}(x) \quad (x \neq (k+\frac{1}{2})\pi, k \in \mathbb{Z}) \Rightarrow \exists\ f'(x) = \frac{1}{\cos^2(x)} \quad (x \neq (k+\frac{1}{2})\pi, k \in \mathbb{Z}) \qquad (\operatorname{tg}'(x) = (\operatorname{tg}(x))' = \frac{1}{\cos^2(x)})$

$f(x) = \operatorname{ctg}(x) \quad (x \neq k\pi, k \in \mathbb{Z}) \Rightarrow \exists\ f'(x) = -\frac{1}{\sin^2(x)} \quad (x \neq k\pi, k \in \mathbb{Z}) \qquad (\operatorname{ctg}'(x) = (\operatorname{ctg}(x))' = -\frac{1}{\sin^2(x)})$

$f(x) = \arcsin(x) \quad (x \in [-1, 1]) \Rightarrow \exists\ f'(x) = \frac{1}{\sqrt{1-x^2}} \quad (x \in]-1, 1[) \qquad (\arcsin'(x) = (\arcsin(x))' = \frac{1}{\sqrt{1-x^2}})$

$f(x) = \arccos(x) \quad (x \in [-1, 1]) \Rightarrow \exists\ f'(x) = -\frac{1}{\sqrt{1-x^2}} \quad (x \in]-1, 1[) \qquad (\arccos'(x) = (\arccos(x))' = -\frac{1}{\sqrt{1-x^2}})$

$f(x) = \operatorname{arctg}(x) \quad (x \in \mathbb{R}) \Rightarrow \exists\ f'(x) = \frac{1}{1+x^2} \quad (x \in \mathbb{R}) \qquad (\operatorname{arctg}'(x) = (\operatorname{arctg}(x))' = \frac{1}{1+x^2})$

$f(x) = \operatorname{arcctg}(x) \quad (x \in \mathbb{R}) \Rightarrow \exists\ f'(x) = -\frac{1}{1+x^2} \quad (x \in \mathbb{R}) \qquad (\operatorname{arcctg}'(x) = (\operatorname{arcctg}(x))' = -\frac{1}{1+x^2})$

$f(x) = \operatorname{arsh}(x) \quad (x \in \mathbb{R}) \Rightarrow \exists\ f'(x) = \frac{1}{\sqrt{x^2+1}} \quad (x \in \mathbb{R}) \qquad (\operatorname{arsh}'(x) = (\operatorname{arsh}(x))' = \frac{1}{\sqrt{x^2+1}})$

$f(x) = \operatorname{arch}(x) \quad (x \in [1, \infty[) \Rightarrow \exists\ f'(x) = \frac{1}{\sqrt{x^2-1}} \quad (x \in]1, \infty[) \qquad (\operatorname{arch}'(x) = (\operatorname{arch}(x))' = \frac{1}{\sqrt{x^2-1}})$

$(x^n)^{(k)} = n(n-1)\cdots(n-k+1)\, x^{n-k} \quad (x \in \mathbb{R})$, ha $k < n$;

$(x^n)^{(n)} = n! \quad (x \in \mathbb{R})$; $\quad (x^n)^{(k)} = 0 \quad (x \in \mathbb{R})$, ha $k > n$

$$\sin^{(n)}(x) = \begin{cases} \cos(x), & \text{ha } n = 4k+1 \\ -\sin(x), & \text{ha } n = 4k+2 \\ -\cos(x), & \text{ha } n = 4k+3 \\ \sin(x), & \text{ha } n = 4k+4 \end{cases} \qquad (k = 0, 1, 2, \ldots)$$